

Lesson 6-7b

Objective - To solve problems involving compound interest.

<u>Simple Interest</u>	<u>Compound Interest</u>
\$100 at 10% for 3 yrs.	\$100 at 10% for 3 yrs. compounded annually
year 1: 10% of 100 = \$10	year 1: 10% of 100 = \$10
year 2: 10% of 100 = \$10	year 2: 10% of 110 = \$11
year 3: 10% of 100 = \$10	year 3: 10% of 121 = \$12.10
<u>Simple Interest = \$30</u>	<u>Compound Interest = \$33.10</u>

Compound Interest = Interest on Principal + Interest on Interest

Types of Compound Interest

Compound Annually = Once per year

Compound Semi-annually = 2 times per year

Compound Quarterly = 4 times per year

Compound Monthly = 12 times per year

Compound Weekly = Once per week (52 times per year)

Compound Bi-weekly = Once every 2 weeks (26 times per year)

Find the total interest earned and the balance of an account that started with \$200 at 5% for 1 year compounded quarterly.

$P = 200$ $r = 5\% = 0.05$ $t = 1$ yr. or 0.25 yrs/qrtr

Quarter 1: $I = P \cdot r \cdot t$ $I = (200)(0.05)(0.25)$ $I = \$2.50$	Quarter 3: $I = P \cdot r \cdot t$ $I \approx (205.03)(0.05)(0.25)$ $I \approx \$2.56$
Quarter 2: $I = P \cdot r \cdot t$ $I = (202.50)(0.05)(0.25)$ $I \approx \$2.53$	Quarter 4: $I = P \cdot r \cdot t$ $I \approx (207.59)(0.05)(0.25)$ $I \approx \$2.59$

Balance = $\$207.59 + 2.59 = \210.18

Total Interest = $\$210.18 - 200 = \10.18

Find the balance of an account holding \$1000 for 2 years compounded semi-annually at 8% annual interest.

$P = 1000$ $r = 8\% = 0.08$ $t = 2$ yrs. or 0.5 yrs/semi

1st 6 months: $I = P \cdot r \cdot t$ $I = (1000)(0.08)(0.5)$ $I = \$40$	3rd 6 months: $I = P \cdot r \cdot t$ $I = (1081.60)(0.08)(0.5)$ $I \approx \$43.26$
2nd 6 months: $I = P \cdot r \cdot t$ $I = (1040)(0.08)(0.5)$ $I = \$41.60$	4th 6 months: $I = P \cdot r \cdot t$ $I \approx (1124.86)(0.08)(0.5)$ $I \approx \$44.99$

Balance = $\$1124.86 + 44.99 \approx \1169.85

Too much work!

Compound Interest Formula

<u>Long Way</u>	<u>Quicker Way</u>
\$100 at 10% for 3 yrs. compounded annually	Start with \$100.
yr. 1: $100 + 10\%$ of 100 $100 + 10 = \$110$	After year 1: 110% of start = \$110
yr. 2: $110 + 10\%$ of 110 $110 + 11 = \$121$	After year 2: 110% of year 1 = \$121
yr. 3: $121 + 10\%$ of 121 $121 + 12.10 = \$133.10$	After year 3: 110% of year 2 = \$133.10

Balance = 110% of 110% of 110% of \$100

Balance = $1.10 \cdot 1.10 \cdot 1.10 \cdot (\$100)$

Balance = $(1.10)^3 \cdot (\$100) = \133.10

Compound Interest Formula

Balance = 110% of 110% of 110% of \$100

Balance = $1.10 \cdot 1.10 \cdot 1.10 \cdot (\$100)$

Balance = $(1.10)^3 \cdot (\$100) = \133.10

$$B = P \cdot \left(1 + \frac{r}{n}\right)^{nt}$$

B = Balance (Total Amount)

P = Principal

$\frac{r}{n}$ = $\frac{\text{Annual Interest Rate}}{\text{\# of compoundings}}$

nt = # of compoundings \times # years

Lesson 6-7b (cont.)

Find the balance in an account that earns 8% annual interest that begins with \$1000 in principal and is compounded semi-annually for 2 years.

$P = 1000$ $r = 8\% = 0.08$ $t = 2$ yrs. Compounded
semi-annually
 $n = 2$

$$B = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$B = 1000\left(1 + \frac{0.08}{2}\right)^{2 \cdot 2}$$

$$B = 1000(1.04)^4$$

$$B \approx \$1169.86$$

An investment of \$4000 compounds quarterly for 3 years at 6% annual interest.

$$P = 4000$$

$$\frac{r}{n} = \frac{0.06}{4} = .015 \text{ (quarterly interest)}$$

$$nt = 4 \cdot 3 = 12 \text{ compounding periods}$$

$$B = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$B = 4000(1 + 0.015)^{12}$$

$$B = 4000(1.015)^{12}$$

$$B \approx \$4782.47$$

Find the balance in each scenario.

$$P = \$500$$

$$r = 8\%$$

$$t = 3 \text{ yrs.}$$

$$B = P\left(1 + \frac{r}{n}\right)^{nt}$$

Compound
Annually

$$\frac{r}{n} = \frac{0.08}{1}$$

$$nt = 3$$

$$B = 500(1 + 0.08)^3$$

$$B \approx \$629.86$$

Compound
Semi-annually

$$\frac{r}{n} = \frac{0.08}{2} = 0.04$$

$$nt = 6$$

$$B = 500(1 + 0.04)^6$$

$$B \approx \$632.66$$

Compound
Quarterly

$$\frac{r}{n} = \frac{0.08}{4} = 0.02$$

$$nt = 12$$

$$B = 500(1 + 0.02)^{12}$$

$$B \approx \$634.12$$