

Lesson 6-4

Objective - To identify the number of solutions a system of linear equations may have.

Solve.

$$\begin{cases} y = -\frac{1}{2}x + 1 \\ x + 2y = 6 \end{cases}$$

$$\begin{array}{r} x + 2y = 6 \\ -x \quad -x \\ \hline 2y = -x + 6 \\ \frac{2y}{2} = \frac{-x + 6}{2} \\ y = -\frac{1}{2}x + 3 \end{array}$$

$m = -\frac{1}{2}$ $m = -\frac{1}{2}$
 $b = 1$ $b = 3$

No Solution

3 Possible Outcomes

1)

Lines Intersect

One Solution

Consistent Independent

2)

Lines Parallel

No Solution

Inconsistent

3)

Lines Coincide

Infinitely Many Solutions

Consistent Dependent

Solve.

$$\begin{cases} y = \frac{1}{3}x + 2 \\ 2x - 6y = -12 \end{cases}$$

$$\begin{array}{r} 2x - 6y = -12 \\ -2x \quad -2x \\ \hline -6y = -2x - 12 \\ \frac{-6y}{-6} = \frac{-2x - 12}{-6} \\ y = \frac{1}{3}x + 2 \end{array}$$

$m = \frac{1}{3}$ $m = \frac{1}{3}$
 $b = 2$ $b = 2$

Infinitely Many Solutions

3 Possible Outcomes

1)

Lines Intersect

One Solution

Different Slopes

2)

Lines Parallel

No Solution

Same Slopes Different y-int.

3)

Lines Coincide

Infinitely Many Solutions

Same Slopes Same y-int.

Without graphing, indicate whether the systems below have one solution, no solution, or infinitely many.

1) $\begin{cases} y = -2x + 1 \\ y = -2x + 5 \end{cases}$

Same Slopes, Different y-int. → Lines are Parallel

No Solution

2) $\begin{cases} y = 3x - 8 \\ y = \frac{1}{3}x + 2 \end{cases}$

Different Slopes → Lines Intersect

One Solution

3) $\begin{cases} 2y - 6 = 4(x + 5) \\ y - 13 = 2x \end{cases}$

$$\begin{array}{r} 2y - 6 = 4(x + 5) \\ y - 13 = 2x \\ \hline 2y - 6 = 4x + 20 \\ +6 \quad +6 \\ \hline 2y = 4x + 26 \\ \frac{2y}{2} = \frac{4x + 26}{2} \\ y = 2x + 13 \end{array}$$

Same Slopes, Same y-int. → Lines Coincide

Infinitely Many Solutions

Solve the system graphically, by substitution, and by elimination.

$$\begin{cases} -x + y = 3 \\ x + 2y = 4 \end{cases}$$

Graphic Method

$\approx \left(-\frac{1}{2}, 2\frac{1}{2}\right)$

Substitution:

$$\begin{array}{r} x + 2y = 4 \\ -x \quad -x \\ \hline 2y = -x + 4 \\ \frac{2y}{2} = \frac{-x + 4}{2} \\ y = -\frac{1}{2}x + 2 \end{array}$$

$m = 1$ $m = -\frac{1}{2}$
 $b = 3$ $b = 2$

Lesson 6-4 (cont.)

Solve the system graphically, by substitution, and by elimination.

$$\begin{cases} -x + y = 3 \\ x + 2y = 4 \end{cases}$$

Substitution

$$\begin{array}{r} -x + y = 3 \\ +x \quad +x \\ \hline y = x + 3 \end{array}$$

$$\begin{array}{r} x + 2y = 4 \\ x + 2(x + 3) = 4 \\ x + 2x + 6 = 4 \\ \hline -6 \quad -6 \\ \hline 3x = -2 \\ \frac{3x}{3} = \frac{-2}{3} \\ x = -\frac{2}{3} \end{array}$$

$$y = x + 3$$

$$y = -\frac{2}{3} + 3$$

$$y = 2\frac{1}{3}$$

$\left(-\frac{2}{3}, 2\frac{1}{3}\right)$

Solve the system graphically, by substitution, and by elimination.

$$\begin{cases} -x + y = 3 \\ x + 2y = 4 \end{cases}$$

Elimination

$$\begin{array}{r} -x + y = 3 \\ x + 2y = 4 \\ \hline 3y = 7 \\ \frac{3y}{3} = \frac{7}{3} \\ y = \frac{7}{3} \end{array}$$

$$\begin{array}{r} x + 2y = 4 \\ x + 2\left(\frac{7}{3}\right) = 4 \\ x + \frac{14}{3} = 4 \\ \hline x = \frac{12}{3} - \frac{14}{3} \\ x = -\frac{2}{3} \end{array}$$

$\left(-\frac{2}{3}, 2\frac{1}{3}\right)$

Solve the system graphically, by substitution, and by elimination.

$$\begin{cases} -x + 2y = 3 \\ 4y = 2x + 8 \end{cases}$$

Graphic Method

$$\begin{array}{r} -x + 2y = 3 \\ +x \quad +x \\ \hline 2y = x + 3 \\ \frac{2y}{2} = \frac{x + 3}{2} \\ y = \frac{1}{2}x + \frac{3}{2} \end{array}$$

$$\begin{array}{r} 4y = 2x + 8 \\ \frac{4y}{4} = \frac{2x + 8}{4} \\ y = \frac{2}{4}x + \frac{8}{4} \\ y = \frac{1}{2}x + 2 \end{array}$$

$m = \frac{1}{2}$ $b = \frac{3}{2}$ $m = \frac{1}{2}$ $b = 2$

No Solution

Solve the system graphically, by substitution, and by elimination.

$$\begin{cases} -x + 2y = 3 \\ 4y = 2x + 8 \end{cases}$$

Substitution

$$\begin{array}{r} -x + 2y = 3 \\ -2y \quad -2y \\ \hline -x = -2y + 3 \\ -1(-x = -2y + 3) \\ x = 2y - 3 \end{array}$$

$$\begin{array}{r} 4y = 2x + 8 \\ 4y = 2(2y - 3) + 8 \\ 4y = 4y - 6 + 8 \\ 4y = 4y + 2 \\ \hline -4y \quad -4y \\ \hline 0 = 2 \quad \text{False!} \end{array}$$

No Solution

Solve the system graphically, by substitution, and by elimination.

$$\begin{cases} -x + 2y = 3 \\ 4y = 2x + 8 \end{cases}$$

Elimination

$$\begin{array}{r} -x + 2y = 3 \rightarrow -2(-x + 2y = 3) \rightarrow 2x - 4y = -6 \\ 4y = 2x + 8 \rightarrow -2x + 4y = 8 \rightarrow -2x + 4y = 8 \\ \hline 0 = 2 \\ \text{False!} \end{array}$$

No Solution