

Lesson 2-4 & 5-3 LR

Objective - To use deductive and inductive reasoning in direct and indirect proofs.

Logical Reasoning

<p><u>Deductive Reasoning</u> - process of demonstrating that the validity of certain statements can imply the validity of statements that follow.</p> <p>All prime numbers greater than 2 are odd. 37 is a prime number. Therefore, 37 is odd.</p>	<p><u>Inductive Reasoning</u> - process of making generalizations based on observed data, patterns, and past performance.</p> <p>You have never seen a pelican in the desert. Therefore, pelicans probably do not live in the desert.</p>
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Conditionals (If-then Statements)

<p><u>Deductive Reasoning</u> If <u>your number is a prime greater than 2</u>, then <u>it is odd</u>.</p> <p><u>Hypothesis:</u> Your number is a prime greater than 2. <u>Conclusion:</u> It is odd. <u>Deduction:</u> Certain! Used in proofs!</p>	<p><u>Inductive Reasoning</u> If <u>you have never seen pelicans in the desert</u>, then <u>they do not live there</u>.</p> <p><u>Hypothesis:</u> You have never seen pelicans in the desert. <u>Conclusion:</u> They do not live there. <u>Induction:</u> Likely! Not often used in proofs!</p>
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Example of Inductive Reasoning

Prove: If one is added to a product of primes, then none of the original primes will be a factor of the new number.

Primes
2, 3, 5, 7, 11, 13, 17, 19...

$2 \cdot 3 + 1 = 7$	$3 \cdot 7 + 1 = 22 = 2 \cdot 11$
$2 \cdot 5 + 1 = 11$	$3 \cdot 11 + 1 = 34 = 2 \cdot 17$
$2 \cdot 7 + 1 = 15 = 3 \cdot 5$	$2 \cdot 3 \cdot 5 + 1 = 31$
$2 \cdot 11 + 1 = 23$	$3 \cdot 5 \cdot 7 + 1 = 106 = 2 \cdot 53$
$2 \cdot 13 + 1 = 27 = 3 \cdot 3 \cdot 3$	The product of x and y plus one will yield a number for which x and y are not factors.
$2 \cdot 17 + 1 = 35 = 5 \cdot 7$	
$2 \cdot 19 + 1 = 39 = 3 \cdot 13$	

Deductive Reasoning

Conjecture - a statement or conditional that one is trying to prove.

Types of supportive statements used in proofs

- 1) Undefined terms - Terminology so fundamental it defies definition.
ie: point, line, straight, etc.
- 2) Definitions - Statements defined by other terms.
ie: A quadrilateral is a 4 sided polygon.
- 3) Axioms (Postulates) - Property or statement which is assumed to be true.
ie: Two points will determine a line.
- 4) Theorems - A property or statement which has been proven to be true.
ie: Pythagorean Theorem $a^2 + b^2 = c^2$.

Additional Properties (Axioms) Used in Proofs

Addition Property of Equality
If $a = b$, then $a + c = b + c$.

Subtraction Property of Equality
If $a = b$, then $a - c = b - c$.

Multiplication Property of Equality
If $a = b$, then $a \cdot c = b \cdot c$.

Division Property of Equality
If $a = b$, then $a \div c = b \div c$.

Deductive Reasoning

<p><u>Direct Proof</u></p> <p>- Usually in two-column format.</p> <p>- Direct proof is made through use of clear statements and properties that follow a logical argument step by step. from hypothesis to conclusion.</p>	<p><u>Indirect Proof</u></p> <p>- Usually in paragraph format.</p> <p>- Indirect proof works in reverse. It attempts to show that if valid reasoning from a hypothesis leads to a false conclusion, then the hypothesis must also be false.</p>
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Lesson 2-4 & 5-3 LR (cont.)

Example of Direct Proof (Deductive)

Prove: If $a = b$, then $-a = -b$.

Statement	Reason
$a = b$	Given
$a + (-b) = b + (-b)$	Addition Property of Equality
$a + (-b) = 0$	Property of Opposites
$(-a) + [a + (-b)] = 0 + (-a)$	Addition Property of Equality
$[(-a) + a] + (-b) = 0 + (-a)$	Associative Prop. of Addition
$0 + (-b) = 0 + (-a)$	Property of Opposites
$-b = -a$	Identity Property of Addition
$-a = -b$	Symmetric Property

Conditional: If $a = b$, then $-a = -b$.

Hypothesis
Conclusion

Example of Direct Proof (Deductive)

Solve the equation. Provide justification for each step.

Statement	Reason
$-3x + 5(4 + x) = 60$	Given
$-3x + 20 + 5x = 60$	Distributive Property
$-3x + 5x + 20 = 60$	Commutative Property of Addition
$2x + 20 = 60$	Combine Like Terms
$2x = 40$	Subtraction Property of Equality
$x = 20$	Division Property of Equality

Conditional: If $-3x + 5(4 + x) = 60$, then $x = 20$.

Hypothesis
Conclusion

State whether the conclusion was reached through deductive or inductive reasoning.

1) All motorcycles have tires. I drive a motorcycle. It must have tires.
Deductive

2) Every motorcycle I have seen has chrome mufflers. If you are going to buy a motorcycle, it will have chrome mufflers.
Inductive

3) Marco notices that the difference between two consecutive perfect squares is always odd. He concludes that the difference between 131^2 and 132^2 must be odd.
Inductive

State whether the conclusion was reached through deductive or inductive reasoning.

4) All of your pets have been furry. You will get a new pet for your birthday. It will be furry.
Inductive

5) A rectangle is a parallelogram with right angles. Every square is a parallelogram with right angles. Every square must also be a rectangle.
Deductive

6) All numbers divisible by 6 are also divisible by 2 and 3. 216 is divisible by 6. Therefore, 216 is divisible by 3.
Deductive

State whether the conclusion was reached through deductive or inductive reasoning.

7) It is always hot in Texas in August. Therefore, if you go to Texas in August, then it will be hot.
Inductive

8) The product of two odd whole numbers is always odd. 31 and 43 are both odd. Therefore, their product will be odd.
Deductive

9) A TV show did an experiment that showed that elephants are afraid of mice. Therefore, the elephants at the zoo will be afraid of mice.
Inductive

Example of Indirect Proof (Deductive)

Prove: The number of primes is infinite.

Suppose that there are a finite number of primes...

$p_1, p_2, p_3 \dots p_n$

Let the product of all of these primes = N

$N = p_1 \cdot p_2 \cdot p_3 \cdot \dots \cdot p_n$

Remember our inductive argument from before, "If one is added to a product of primes, then none of the original primes will be a factor of the new number."

We know that $N+1$ is not divisible by any of the primes used to make N. Therefore, $N+1$ must be a new prime number not in our set of finite primes.

Hence, the supposition is false. There can't be a finite number of primes. The number of primes must be infinite.

Lesson 2-4 & 5-3 LR (cont.)

Another Example of Indirect Proof (Deductive)

Prove: $\sqrt{2}$ is an irrational number.

Definition: An irrational number is any real number which can't be written as a fraction,

$\frac{p}{q}$, where p and q are integers.

Suppose $\sqrt{2} = \frac{p}{q}$

$$(\sqrt{2})^2 = \left(\frac{p}{q}\right)^2$$

$$(q^2)2 = \frac{p^2}{q^2} (q^2)$$

Another Example of Indirect Proof (Deductive)

Prove: $\sqrt{2}$ is an irrational number.

$$2q^2 = p^2$$

Here lies the contradiction. Integers p and q are prime or are a product of primes .

$$p = x_1 \cdot x_2 \cdot x_3 \cdot \dots \cdot x_n$$

$$q = y_1 \cdot y_2 \cdot y_3 \cdot \dots \cdot y_n$$

If $2q^2 = p^2$,

$$\text{then } 2(x_1 \cdot x_2 \cdot x_3 \cdot \dots \cdot x_n)^2 = (y_1 \cdot y_2 \cdot y_3 \cdot \dots \cdot y_n)^2.$$

**Even number
of primes.**

**Even number
of primes.**

Another Example of Indirect Proof (Deductive)

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$$\text{then } 2(x_1 \cdot x_2 \cdot x_3 \cdot \dots \cdot x_n)^2 = (y_1 \cdot y_2 \cdot y_3 \cdot \dots \cdot y_n)^2.$$

**Even number
of primes.**

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There is no factor that pairs with the two.

It is impossible to have an odd number of prime factors on the right side of an equation and an even number of prime factors on the left.

Therefore, $2q^2 \neq p^2$ and, $\sqrt{2} \neq \frac{p}{q}$.